

Induced Gravity from Theory Space

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Abstract

The mass spectrum of a model constructed in a theory space is expressed by eigenvalues of the Laplacian on the graph structure of the theory space. The nature of the one-loop UV divergence in the vacuum energy is then controlled only by the degree matrix of the graph. Using these facts, we can construct models of induced gravity which do not suffer from divergences at the one-loop level.

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I. INTRODUCTION

The idea of dimensional deconstruction [1] has appeared recently. A number of copies of a four-dimensional ‘theory’ as well as a new set of fields linking pairs of these ‘theories’ are considered. Then the resulting whole theory given by the ‘theory space’ may be equivalent to a higher-dimensional theory with discretized extra dimensions.

The simplest model with the Abelian symmetry is studied by Hill and Leibovich [2]. The lagrangian density for vector fields is written as

$$\mathcal{L}_A = \frac{1}{g^2} \sum_{k=1}^N \left[-\frac{1}{4} F_k^{\mu\nu} F_{k\ \mu\nu} - (D^\mu U_k)^\dagger D_\mu U_k \right], \quad (1)$$

where g is a gauge coupling, $F_k^{\mu\nu} = \partial^\mu A_k^\nu - \partial^\nu A_k^\mu$ and $\mu, \nu = 0, 1, 2, 3$. We should read $A_{N+1}^\mu = A_1^\mu$, *etc.* The link fields U_k are transformed as $U_k \rightarrow W_k U_k W_{k+1}^*$ ($|W_k| = 1$ for any k). The covariant derivatives are defined as $D^\mu U_k = \partial^\mu U_k - i A_k^\mu U_k + i U_k A_{k+1}^\mu$.

Now we assume that the absolute value of each link field $|U_k|$ has a common value $f/\sqrt{2}$. Then the part of the lagrangian for the link fields gives a mass term for the vector fields in the background field: For example, if $N = 5$, the $(mass)^2$ matrix takes the form[15]

$$f^2 \begin{pmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{pmatrix}. \quad (2)$$

As seen later, up to the dimensional coefficient f^2 , this matrix is identified with the Laplacian matrix for the graph C_N , the cycle graph with N vertices. We find, indeed, any theory space can be associated with the graph if the link fields have a common value.

Similarly, field theory on the space as a discrete set of points has been studied by many authors [3]. We use the ‘graph theory space’ as an extra space, so we do not worry about the naive continuous limit; but we study the UV divergent behavior of the four-dimensional one-loop effective lagrangian.

In Sec. II, we study the Laplacian of the graph theory space. The one-loop effective lagrangian density is discussed in Sec. III. We construct models of induced gravity Sec. IV, by using several graphs. We close with Sec. V, where summary and prospects are given.

II. LAPLACIAN FOR GRAPH THEORY SPACE

In this section, after a brief description of graph theory [4], the Laplacian for a graph is introduced.[16] Suppose $G = (V, E)$ is a simple graph, where V is the set of vertices while E is the set of edges connecting two vertices. Here ‘simple’ means that the graph does not include multiple edges and self-loops. The order of G , denoted by p , is the number of vertices in the graph and the size of G , denoted by q , is the number of edges in the graph.

A pair of vertices v_i and v_j are said to be adjacent if there exists an edge $e \in E$ such that $e = \{v_i, v_j\}$. The adjacency matrix $A(G) = (a_{ij})_{v_i, v_j \in V}$ is defined as

$$a_{ij} = \begin{cases} 1 & : \text{if } v_i \text{ is adjacent to } v_j \\ 0 & : \text{otherwise} \end{cases} . \quad (3)$$

The degree of a vertex $\deg(v)$ is the number of edges directly connected to v . The (diagonal) degree matrix $D(G) = (d_{ij})_{v_i, v_j \in V}$ is defined as

$$d_{ij} = \begin{cases} \deg(v_i) & : \text{if } v_i = v_j \\ 0 & : \text{otherwise} \end{cases} . \quad (4)$$

Then we define the Laplacian matrix of G as

$$\Delta(G) = D(G) - A(G) . \quad (5)$$

The $(mass)^2$ matrix for vector fields in the Hill-Leibovich model [2]((2) for $N = 5$) is written as $f^2\Delta(C_N)$, where C_N is the cycle graph with N vertices (FIG. 1), because $D(C_N) = \text{diag. } (2, 2, \dots, 2)$ and only non-zero elements of $A(C_N)$ are $a_{i, i \pm 1}$ (where the suffices should be read in modulo N).

The lagrangian for the vector fields can be naturally constructed by putting the $D_\mu U D^\mu U$ terms on the corresponding edges. We must put plaquette type terms to give masses to the neutral scalar fields originated from the link fields [5]. In this paper, we concentrate the case with $p = q$ (which holds for $G = C_p$). Then we have only one massless scalar field in addition to the vector fields.

On the other hand, for elementary scalar fields, it is easy to write the lagrangian such as $f^2\phi_i\Delta_{ij}\phi_j$, as long as gauge symmetry is omitted. The link fields can be attached according to the symmetry of each theory on the vertex, such as $\phi_i\phi_j \Rightarrow |U_{\{i,j\}}|^{-1}\phi_i U_{\{i,j\}}\phi_j$, where the gauge transformations $\phi_k \rightarrow W_k\phi_k$ and $U_{\{i,j\}} \rightarrow W_i U_{\{i,j\}} W_j^\dagger$ are imposed.

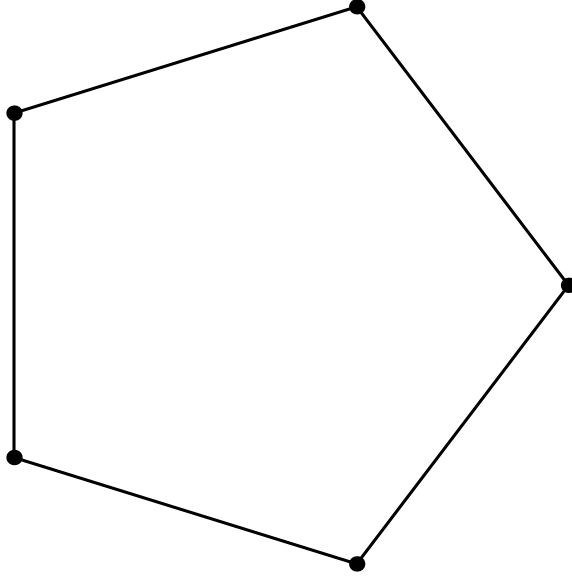


FIG. 1: C_5 . $p = q = 5$ for this graph.

For the Dirac fields, Hill and Leibovich considered the Wilson fermion on C_N [2]. Here we suppose the simple case with $U_k = f/\sqrt{2}$. Then the lagrangian for the Wilson fermion with a unit hopping parameter takes the form

$$\begin{aligned} \mathcal{L}_\psi = & \sum_{k=1}^N \left[i\bar{\Psi}_{kL} \not{\partial} \Psi_{kL} + i\bar{\Psi}_{kR} \not{\partial} \Psi_{kR} \right] \\ & + \sum_{k=1}^N f \left[\bar{\Psi}_{kL} (\Psi_{kR} - \Psi_{k+1,R}) + \text{h.c.} \right] . \end{aligned} \quad (6)$$

If we wish to generalize the lagrangian to the case with an arbitrary graph of $p = q$, we first introduce the incidence matrix $B(G) = (b_{ij})_{v_i \in V, e_j \in E}$ defined as

$$b_{ij} = \begin{cases} 1 & : \text{ if the edge } e_j \text{ is connected to the vertex } v_i \\ 0 & : \text{ otherwise} \end{cases} . \quad (7)$$

By rearranging the label of vertices, we make the diagonal elements of B unity. Let E be the matrix whose elements are equal to B but off-diagonal elements change the sign. E is often quoted as the incidence matrix for the directed graph in some literature. The following relation is known:

$$EE^T = \Delta(G) . \quad (8)$$

We can write the mass matrix part of the lagrangian in general, as

$$f \left[\bar{\Psi}_{iL} E_{ij}^T \Psi_{jR} + \text{h.c.} \right] . \quad (9)$$

The inclusion of the link variable is possible; we will see that later in the certain model and more rigorous treatment will be found in [6].

Now the Dirac fermions on G have the $(mass)^2$ matrix $f^2 \Delta(G)$ as in the case for the bosonic fields. Only two comments are in order here: First, for models deconstructing higher dimensions, we consider some Cartesian product of graphs. Second, by construction, we find that one chirality of fermions are *naturally* associated with edges (not with vertices), and Eq. (8) holds even for $p \neq q$. [17]

Many interesting results on the graph theory have been found in the mathematical literature. The spectrum of the Laplacian for graphs is studied in the mathematics community. [18] It is straightforward to figure out the mass spectrum of the field theory in theory space of a general graph by using the mathematical results. [19] In the rest of the present paper, however, we will concentrate on the study of one-loop UV divergence of the model in the graph theory space. We leave the estimation of the spectrum and the physical discussion of the observability of excited modes for future work [6].

III. ONE-LOOP EFFECTIVE LAGRANGIAN

For a gaussian scalar field in the graph theory space G , the partition function is defined as

$$Z = \int \mathcal{D}\phi \exp \left[-\frac{1}{2} \phi \left(-\nabla^2 + \Delta(G) \right) \phi \right] , \quad (10)$$

where we omitted the indices of scalars as well as the integration on the spacetime ($\int d^4x$), and ∇^2 is the usual laplacian for the scalar field. Further, we set the factor of scale f to unity for simplicity. Performing the gaussian integral, we obtain

$$\begin{aligned} Z &\propto \left[\det \left(-\nabla^2 + \Delta(G) \right) \right]^{-1/2} = \exp \left[-\frac{1}{2} \ln \det \left(-\nabla^2 + \Delta(G) \right) \right] \\ &= \exp \left[-\frac{1}{2} \text{tr} \ln \left(-\nabla^2 + \Delta(G) \right) \right] . \end{aligned} \quad (11)$$

The argument of the exponential in the right hand side is the one-loop effective action. Using the Schwinger parameter, one can find

$$\ln \left(-\nabla^2 + \Delta(G) \right) = - \int_{1/\Lambda^2}^{\infty} \frac{dt}{t} \exp \left[- \left(-\nabla^2 + \Delta(G) \right) t \right] , \quad (12)$$

where Λ is the UV cutoff scale.

In the flat d -dimensional spacetime, the part including the d -dimensional laplacian becomes

$$\text{tr} \exp \left[- \left(-\nabla^2 \right) t \right] = \int \frac{d^d p}{(2\pi)^d} \exp \left[-p^2 t \right] = \frac{1}{(4\pi)^{d/2}} t^{-d/2}. \quad (13)$$

On the other hand, we find

$$\text{Tr} \exp \left[-\Delta(G)t \right] = p - (\text{Tr} D) t + \frac{1}{2} \left[\text{Tr} D^2 + \text{Tr} D \right] t^2 - + \cdots, \quad (14)$$

since $\text{Tr} A^2 = \text{Tr} D$ holds for an arbitrary simple graph.

From (13) and (14), we find that the UV divergent terms depend only on the degree matrix D (and the order p) for $d = 4$. In particular, the theory on the graph with the same degree matrix, even if its adjacency matrix is different, has the same one-loop divergences in vacuum.[20]

We consider the case that the appropriate link fields are associated and a zero-mode field δ is survived as $U \propto \exp(i\delta)$. The corresponding adjacency matrix A has the non-zero element a_{ij} only for the adjacent pair v_i, v_j , but in this case the value is $\exp(\pm i\delta)$ instead of unity. In addition, A is an hermite matrix. Even in this case, $\text{Tr} A^2 = \text{Tr} D$ holds, therefore the divergent term is unchanged; in other words, the one-loop effective potential for the zero-mode of the link field can be defined unambiguously up to the constant.

Thus the zero-mode field of the link variable acquires mass by the one-loop quantum effect. We can consider the phase variable as a constant instead of a dynamical variable. Then the mechanism of generation of mass can be regarded as a generalization of the Scherk-Schwarz mechanism [9], since the theory on the graph C_N in the limit of $N \rightarrow \infty$ corresponds to its compactification on S^1 .

If some non-Abelian gauge symmetry is introduced, a symmetry breaking mechanism becomes possible in the theory on a graph, just as in the case of the Hosotani mechanism [10]. This generalization of the Hosotani mechanism will be studied in future work.

IV. ONE-LOOP FINITE NEWTON CONSTANT FROM GRAPH THEORY SPACE

If the spacetime is curved, the scalar curvature arises in the effective lagrangian. Then we have the Einstein-Hilbert action as a quantum effect. Such an ‘induced gravity’ scenario

has been studied since nineteen-sixties [11]. For a recent review, see [12].

In the curved space, we find

$$\text{tr exp} \left[- \left(-\nabla^2 \right) t \right] = \int \frac{d^d p}{(2\pi)^d} \exp \left[-p^2 t \right] = \frac{\sqrt{g}}{(4\pi)^{d/2}} t^{-d/2} (a_0 + a_1 t + \dots), \quad (15)$$

where a_0, a_1, \dots depends on the background fields. For a minimal scalar field, $a_0 = 1$ and $a_1 = (1/6)R$, where R is the scalar curvature. For a Dirac spinor field, $a_0 = 2^{[d/2]}$ and $a_1 = -2^{[d/2]}(1/12)R$. At the same time, we also remember that the fermion loop demands an extra minus sign. For a massive vector field, $a_0 = d - 1$ and $a_1 = [-1 + (d - 1)/6]R$. Then the effective lagrangian takes the form

$$-V_0 + \frac{1}{16\pi G_N} R + \dots, \quad (16)$$

where V_0 stands for the vacuum energy density and G_N is the induced Newton constant.

If we want to calculate the Newton constant and the cosmological constant (which is often defined as $8\pi G_N V_0$) unambiguously at one-loop level, we should consider the cancellation of the divergence. Because the one-loop contribution of bosonic and fermionic fields have different signs, the cancellation is possible if we choose the matter content of the field theory. The complete cancellation of divergence can be expected if the theory has supersymmetry. In such a case, however, it is difficult to obtain a finite value of the Newton constant.

In the last section, we saw that the one-loop divergences are governed by the degree matrix of the graph in the theory on the graph. Even if the individual theory on the vertex is identical, the different graphs for the different fields are possible. We can choose the graphs and matter content to cancel the one-loop divergences.[21]

For example, we will investigate a simple case. Consider a disconnected graph $G = G_1 \cup G_2 \cup \dots \cup G_s$ and $G_i \cap G_j = \phi$ (the empty set) for any $i, j = 1, 2, \dots, s$. Then let $G_i = C_{p_i}$, where $p_1 + p_2 + \dots + p_s = p$. Any choice of p_i yields the same degree matrix *diag.* $(2, 2, \dots, 2)$. Since p' is equal to or more than three for a $C_{p'}$, the total order p is equal to or more than six. Thus the appropriate choice of matter content bring about the cancellation of divergences among the different graphs.

Further in this section, we exhibit minimal models with $p = 6$ explicitly.

In the first case, we consider a vector field theory on $G_V = C_6$, which is described by the lagrangian (1). We take the zero-mode of the link field into account here, so we set $U = (f/\sqrt{2}) \exp(i\delta)$ for all of the link fields associated with edges. A Dirac field theory is

assumed on the same $G_F = G_V = C_6$. We try to consider the coupling between the Dirac field and the link field. According to [2], the kinetic term on each vertex and the ‘mass term’

$$f \sum_k \left[\bar{\Psi}_{kL} \left(\Psi_{kR} - e^{i\delta} \Psi_{k+1,R} \right) + \text{h.c.} \right], \quad (17)$$

is required in the lagrangian if only the link zero-mode is taken account. In addition, a real scalar field on a disconnected graph $G_S = C_3 \cup C_3$ is introduced. The ‘mass term’ looks like

$$f^2 \phi_i \Delta(G_S)_{ij} \phi_j = \frac{f^2}{2} \left[\sum_{i=1}^3 \sum_{j=1}^3 (\phi_i - \phi_j)^2 + \sum_{i=4}^6 \sum_{j=4}^6 (\phi_i - \phi_j)^2 \right], \quad (18)$$

in the present case.

In this model, the one-loop vacuum energy density is calculated as

$$\begin{aligned} V_0 = & + \frac{3}{4\pi^2} \left(\frac{f}{6} \right)^4 \sum_{q=1}^{\infty} \frac{4 \cos(6q\delta) - 3}{q(q^2 - 1/36)(q^2 - 1/9)} \\ & - \frac{24}{\pi^2} \left(\frac{f}{6} \right)^4 \sum_{q=1}^{\infty} \frac{1}{q(q^2 - 1/9)(q^2 - 4/9)}, \end{aligned} \quad (19)$$

while the induced Newton constant becomes

$$\begin{aligned} \frac{1}{16\pi G_N} = & + \frac{1}{48\pi^2} \left(\frac{f}{6} \right)^2 \sum_{q=1}^{\infty} \frac{2 \cos(6q\delta) - 3}{q(q^2 - 1/36)} \\ & + \frac{1}{6\pi^2} \left(\frac{f}{6} \right)^2 \sum_{q=1}^{\infty} \frac{1}{q(q^2 - 1/9)}. \end{aligned} \quad (20)$$

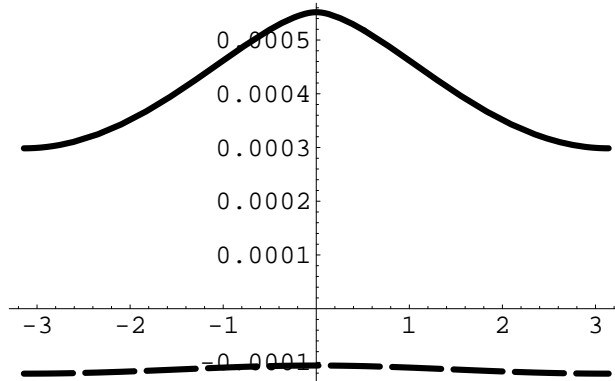


FIG. 2: V_0 (the dashed line) and $1/(16\pi G_N)$ (the bold line) in the first model.

The vacuum energy density and the Newton ‘constant’ are plotted against the link zero-mode in FIG. 2 for this model. We set $f = 1$. The horizontal axis indicates the value 6δ .

The bold line shows $1/(16\pi G_N)$ and the dashed line shows V_0 . We see that G_N is finite and positive while V_0 is finite but negative.

Now we consider another model. The vector field is considered on $G_V = C_6$ as in the first model, but the Dirac field is on a disconnected graph $G_F = C_3 \cup C_3$. Although the coupling between the Dirac field and the link field is arbitrarily selected, it is natural to assume that two edges of each C_3 is connected by the link field. Then the phases from the link zero-mode are attached only to such edges. In FIG. 3, we exhibit the coupling to the link fields. The real scalar field is considered to be on a disconnected graph $C_3 \cup C_3$ as before.

In this model, the one-loop vacuum energy density is calculated as

$$V_0 = - \frac{9}{4\pi^2} \left(\frac{f}{6}\right)^4 \sum_{q=1}^{\infty} \frac{1}{q(q^2 - 1/36)(q^2 - 1/9)} \\ + \frac{24}{\pi^2} \left(\frac{f}{6}\right)^4 \sum_{q=1}^{\infty} \frac{4 \cos(2q\delta) - 1}{q(q^2 - 1/9)(q^2 - 4/9)}, \quad (21)$$

while the induced Newton constant is

$$\frac{1}{16\pi G_N} = - \frac{1}{16\pi^2} \left(\frac{f}{6}\right)^2 \sum_{q=1}^{\infty} \frac{1}{q(q^2 - 1/36)} \\ + \frac{1}{6\pi^2} \left(\frac{f}{6}\right)^2 \sum_{q=1}^{\infty} \frac{2 \cos(2q\delta) + 1}{q(q^2 - 1/9)}. \quad (22)$$

In FIG. 4, we exhibit the vacuum energy density and the Newton ‘constant’ for the second model. Each line shows the same as in the previous figure, but the horizontal axis indicates the value 2δ in the present case. We see that both G_N and V_0 have definite sign. Note that $G_N > 0$ at the point satisfying $V_0 = 0$.

We should emphasize that these models are only minimal models. Even if the matter content is fixed, the choice of graphs is arbitrary; we can search the adequate graphs yielding the small cosmological constant. But the possibility may be constrained by the requirement for the excited modes of fields to decouple from the low energy phenomenology. Roughly speaking, the total degree of a graph is larger the first excited mode is more massive. Obviously, however, the number of the graphs with the common degree matrix will decrease if the order of the graph is fixed. Thus the possible graph may be restricted by the physical reason. The detailed discussion including the general graph spectrum will be given in the future work.

One more remark may be necessary. Besides we can incorporate the known tuning mechanisms of the cosmological constant so far, we can consider a new scenario taking the link

zero-mode field for a time-dependent field. In the second minimal model shown above, the vacuum energy can be small if the zero-mode field takes a certain value. Because this cannot be realized as a static solution, we must consider dynamical evolution of the field in the similar possible models, such as the mechanism suggested by Linde [14].

V. SUMMARY AND PROSPECTS

In conclusion, we clarified the UV behavior of the one-loop effective lagrangian in the theory on graphs. Using this knowledge, we can build a theory space description of the induced gravity. More elaboration is in order for a small cosmological constant.

We must study the following generalization.

We should consider more general case with the graph of $p \neq q$. The gauge theory on such a graph will require plaquette-like self-interaction of link fields. In the same case, probably we cannot avoid the fermion (and chiral fermion zero-modes) on edges. We may have more than the ‘copies’ of the theory on vertices plus link fields on edges. But we should remember that large arbitrariness is dangerous in building plausible models.

To consider the generalization of the Hosotani model, we should investigate deconstruction of non-Abelian gauge theory. We also need deconstruction of adjoint matter fields or fields in other representations.

In this paper, we assumed only one mass scale (f). If several mass scales are associated to edges, the corresponding graph has weighted edges.

We are interested also in the two-loop effective action. We hope that the knowledge of graphs may be useful to investigate higher-loop divergence as well as tree-level calculation of reaction amplitude mediated by the excited modes.

Acknowledgments

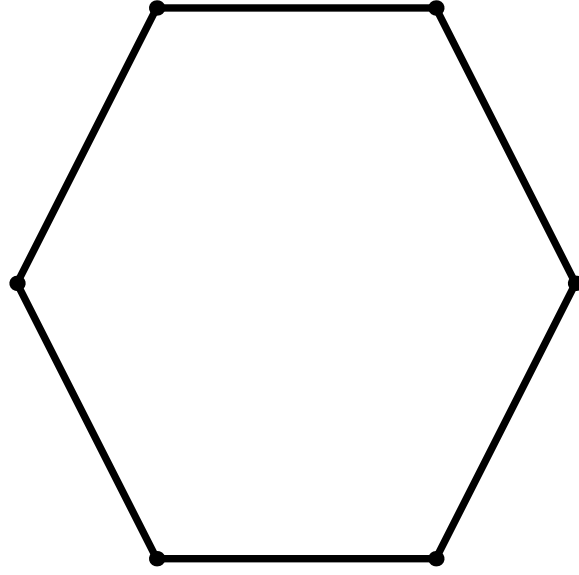
We would like to thank Y. Cho for his valuable comments and for reading the manuscript.

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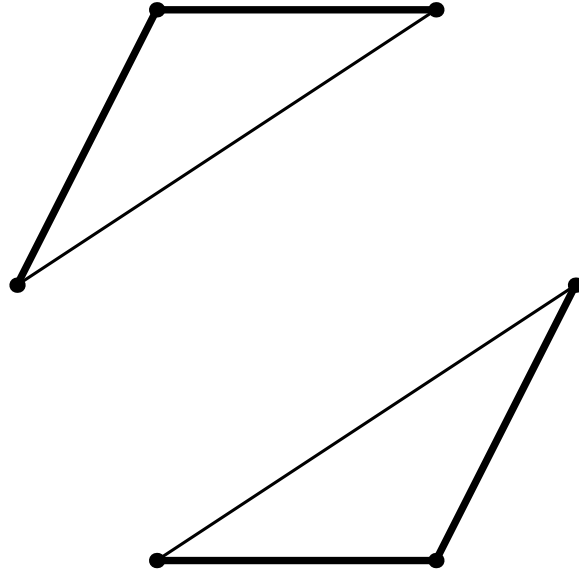
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- [15] As explained in [2], the massive modes absorb the modes of the link field, and the zero-mode of the link field survives.
- [16] More discussion on the mathematical description will be found in [6].
- [17] Eventually, Arkani-Hamed *et al.* argued that the link field originates from the condensation of fermions. We do not pursue something about this scenario here.
- [18] See [8] and references therein. Note however that this book mainly treat the spectrum of another but related Laplacian.
- [19] In general Kaluza-Klein theories, the bound for the first excited mass level is discussed in [7].
- [20] In general, of course, the same divergences can be found in the case with the common values of $\text{Tr}D$ and $\text{Tr}D^2$ respectively and D itself is not necessarily identical. For example, $D(G_1) = \text{diag. } (1, 1, 2, 2, 2, 2, 2, 4)$ and $D(G_2) = \text{diag. } (1, 1, 1, 2, 2, 3, 3, 3)$ satisfy the condition $\text{Tr}D(G_1) = \text{Tr}D(G_2)$ and $\text{Tr}D^2(G_1) = \text{Tr}D^2(G_2)$ (and $p = q$).
- [21] The induced gravity models were considered in the context of the Kaluza-Klein theories [13]. The higher-dimensional divergences are regularized but the treatment of them is unclear in a certain sense.



(G_V)



(G_F)

FIG. 3: G_V and G_F in the second minimal model. The bold edges indicate the location of the link field (zero-mode).

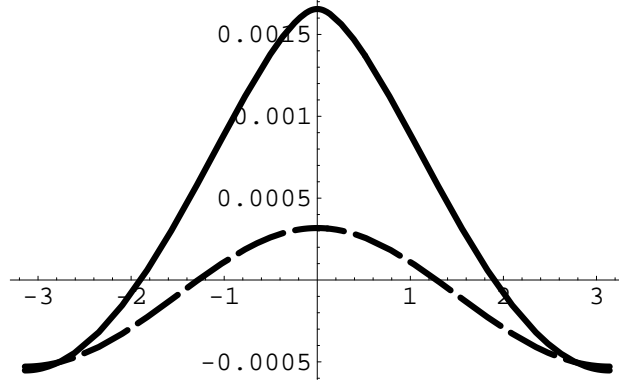


FIG. 4: V_0 (the dashed line) and $1/(16\pi G_N)$ (the bold line) in the second model.